

Mathemania - 2020 Tintin and the Secret of Unicorn

Level 2 Barnaby's Assassination

After winning the game against Ivanovich, Tintin finally gets the Unicorn to his hotel, Sri Nivasa. Soon after, he gets a call from a man named Barnaby. Barnaby claims that the model of the Unicorn has a secret hidden in it and that Ivanovich has planned an attempt to steal the Unicorn. He requests a meeting with Tintin. Curious to know the reason behind Ivanovich's desperation to get the Unicorn, Tintin agrees to meet Barnaby. The doorbell rings at the pre-decided time, but as soon as Tintin opens his door, a gun shot is fired on Barnaby and he falls dead at Tintin's door. The gunman who was in the corridor runs away.

2.1 Problem 1

To investigate the curious incidents, Tintin calls up his detective friends Thompson and Thomson. As they meet up, they have the following conversation:

Tintin: How many criminals have you caught till date?

Thomson: I can tell you but I won't tell Thompson.

Thompson: So can I and neither would I tell Thomson.

So both of them separately whisper the number of criminals each has caught in Tintin's ear, careful that the other doesn't listen.

Tintin: Both of you have caught at least one criminal but one of you has caught one more than the other.

Thomson: I have no idea if you have caught more criminals than I did.

Thompson: Me neither. Do you know now?

Thomson: Yes, indeed!

Thompson: Really? Then so do I!

Note: They have not caught any criminal together (i.e. no criminal is common for both of them)

<u>Submission Guideline</u>: If there are p possible values for the number of criminals Thomson has caught, then submit the sum of all the p values. If p = 0 (i.e. no possible case), then submit 0.

2.2 Problem 2

Done with the small talk, Thompson and Thomson finally begin the investigation. Initially, all the n people who live in the locality are considered equally likely to have shot Barnaby. However, the old lady from the adjacent room (also an eyewitness to the entire incident) reports that the assassin had 6 fingers on his right hand.

Let x and y be the respective probabilities that an innocent man has six fingers on his right hand and that the assassin has six fingers on his right hand. Note that x < y and y may be less than 1 as old eye witnesses are not entirely reliable.

It is found that Ivanovich, who lives in the locality, has 6 fingers in his right hand. What is the probability

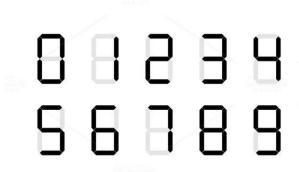
that Ivanovich is the assassin?

<u>Note</u>: Thompson and Thomson have no prior knowledge about Ivanovich and his curiosity in the ship. Hence, this being a Math question, does not take into consideration any qualitative premonitions about Ivanovich

<u>Submission Guideline</u>: Submit the expression in terms of x and y. Please use brackets extensively, as any confusion in syntax would lead to rejection of solution.

2.3 Problem 3

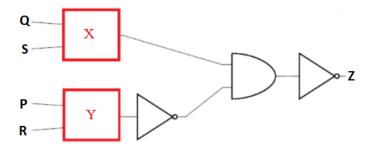
Shaken by the killing of Barnaby, Tintin decides to design a digital lock for protecting the Unicorn. The digital lock has a display which uses digital numbers of the kind shown below-



While constructing the display, he got confused about the logic gates leading to this portion of the circuit marked here-



He has designed the following circuit of logic gates. The decimal digit to be represented is first converted into binary, with 4 binary digits P, Q, R, S. These are the input to the circuit. Can you help Tintin complete the circuit by determining which logic gate would take place of the boxes labelled X and Y.



Submission Guideline: Submit answer as (X,Y) with X and Y as the names of the logic gate.

2.4 Problem 4

To further protect the Unicorn, Tintin decides to setup a laser security system in his hotel courtyard.

The courtyard is a 10 metres \times 10 metres square, covered with 100 square tiles each of 1 metre \times 1 metre A single laser emitter occupies one tile and emits lasers along the two diagonals of the square (as shown in figure below). Hence, if there is any obstruction on any tile located on the diagonals of the laser-emitter-tile, an alarm would be triggered. What are the maximum number of laser emitters that Tintin can place in this courtyard?

<u>Note</u>: No laser emitter may be placed on the tiles secured by another laser emitter as that would be considered as an obstruction and would perpetually buzz the alarm. However, a tile may be secured by multiple laser emitters.

Submission Guideline: Submit the maximum number of laser emitters.

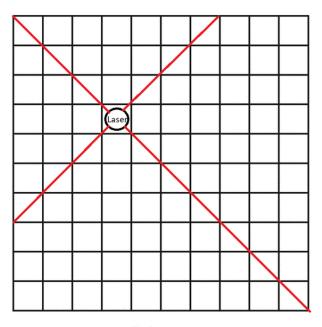
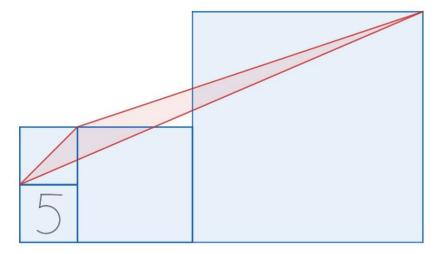


Fig 2a

2.5 Problem 5

Tintin then decides to step up the security of the hotel lobby, by installing landmines. He comes up with a floor-plan to lay down the mines. The floor-plan looks as follows:



Tintin knows that the area of his room (the bottom left square is 5 meter squares.) If the cost of laying down a mine is 10 Eulor/meter-square, what is the amount Tintin will have to spend in order to mine the red area?

Submission Guideline: Submit the amount required, along with explanation.